SOLITONS WITH A SUBSTANTIALLY NONLINEAR ONE-DIMENSIONAL CHAIN

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The investigation of the dynamics of a number of structures, particularly the heat exchangers of electric stations, results in the barely studied computational schemes with many impact pairs [1, 2] that do not allow direct application of traditional methods of analyzing vibration impact systems [3]. Utilization of the analytical approximation of shock interaction permits reduction of the initial problem, under specific conditions, to the computation of a mechanical system (beam, string) with nonlinear elastic supports. Consequently, certain general regularities of the dynamical behavior of such systems, associated with the existence of regimes of soliton type therein [4], are successfully clarified. This paper is devoted to a detailed numerical investigation of such regimes in the simplest system of the class under consideration, a one-dimensional chain of masses connected by means of a weightless string and interacting with strongly nonlinear elastic supports.

Let us note that chains with longitudinal exponential interaction [5, 6] have been studied well at this time. Anharmonic chains on an elastic basis were studied in a long-wavelength approximation [7].

1. The equations of motion of the system under consideration have the form

$$m\ddot{\mathbf{w}}_j + c(2\mathbf{w}_j - \mathbf{w}_{j-1} - \mathbf{w}_{j+1}) + 2n\dot{\mathbf{w}}_j + F(\mathbf{w}_j) = 0, \quad (1.1)$$

where $\mathbf{w}_j$ is a vector with the components $w^{(1)}_j$, $w^{(2)}_j$; m is the magnitude of each of the concentrated masses; c is the stiffness of a linear spring connecting two successive masses. If S is the string tension and l the spacing between the masses, then the stiffness is $c = S/l$. For infinite strings $j = \ldots -2, -1, 0, 1, 2, \ldots$; in the case of a finite length $j = 1, 2, \ldots, N$, where $\mathbf{w}_0 = \mathbf{w}_{N+1}$ in conformity with the conditions for fastening the string in the transverse direction. The nonlinear function $F(\mathbf{w}_j)$ that describes interaction of impact type (abrupt rise in reaction for definite magnitudes of the displacements) is given as follows:

$$F(\mathbf{w}_j) = a\frac{\mathbf{w}_j}{|\mathbf{w}_j|}\operatorname{sh} b\,|\mathbf{w}_j|, \quad a > 0, \quad b > 0. \quad (1.2)$$

Certain typical characteristics of impact pairs in vibrational impact systems cited in [3], say, can be approximated by (1.2) (for an appropriate selection of a and b).

The equations of motion of an infinite chain allow exact periodic solutions in the form of standing waves

$$w_j^{(1)} = (-1)^j\varphi(t), \quad w_j^{(2)} = \alpha w_j^{(1)},$$

where the function $\varphi(t)$ satisfies the ordinary differential equation (α is any real number)

$$\ddot{\varphi} + 4\frac{c}{m}\varphi + 2\frac{n}{m}\dot{\varphi} + \frac{a}{m\sqrt{1+\alpha^2}}\operatorname{sh} b\sqrt{1+\alpha^2}\varphi = 0.$$

A soliton can be represented in the form of a modulated wave

$$w_j^{(1)} = (-1)^j v_j^{(1)}(t), \quad w_j^{(2)} = \alpha w_j^{(1)}(t),$$

where the functions $v^{(1)}_j(t)$ are not identical for different subscripts. If the set of functions $v^{(1)}_j(t)$ describing the modulation depends smoothly on the subscript j, it can be replaced approximately by a function of two variables v(x, t) for which, after the change of variables

$$u = \sqrt{1+\alpha^2}bv, \quad \tau = (\sqrt{ab/m})t, \quad \xi = (\sqrt{ab/cl^2})x,$$

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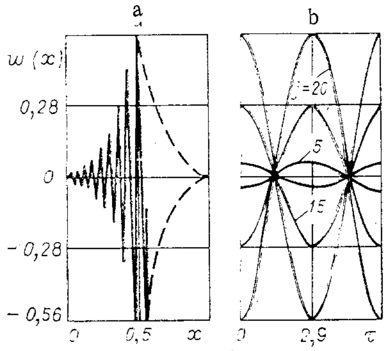

Fig. 1

the following partial differential equation can be written

$$\partial^2 u/\partial\tau^2 + \partial^2 u/\partial\xi^2 + r^2 u + 2\delta\dot{u} + \operatorname{sh} u = 0, \tag{1.3}$$

where $r^2 = 4c/\alpha b$ and $\delta = n/\sqrt{\alpha bm}$. For $\delta = 0$ and $r^2 << 1$, Eq. (1.3) has a solution in the form of a localized standing wave, a soliton of the envelope [4]

$$u(\xi, \tau) = 4\operatorname{arth}\left[\frac{\sqrt{\omega^2 - 1}\cos\omega\tau}{\omega\operatorname{ch}\left(\sqrt{\omega^2 - 1}\,\tau\right)}\right], \tag{1.4}$$

where

$$w_j^{(1)}(\tau) = (-1)^j \frac{1}{b\sqrt{1+\alpha^2}} u(\xi_j, \tau),\quad w_j^{(2)}(\tau) = \alpha w_j^{(1)}(\tau). \tag{1.5}$$

The parameter ω characterizing the frequency of vibrations in the variable τ, determined the wave amplitude and its degree of spatial localization at the same time. As ω increases the soliton becomes so narrow that the conditions for applicability of the long-wave approximation are disturbed.

In order to reduce the constraints on the soliton profile, the localized standing waves were investigated numerically on the basis of the initial system of equations (1.1), which was converted by the change of variables

$$\mathbf{u}_j = b\mathbf{w}_j,\quad \tau = (\sqrt{ab/m})t$$

to the form

$$\frac{d^2\mathbf{u}_j}{d\tau^2} + \beta(2\mathbf{u}_j - \mathbf{u}_{j+1} - \mathbf{u}_{j-1}) + 2\delta\dot{\mathbf{u}}_j + \frac{\mathbf{u}_j}{|\mathbf{u}_j|}\operatorname{sh}|\mathbf{u}_j| = 0, \tag{1.6}$$

where $\beta = r^2/4$.

The quantity of masses for numerical integration was taken at 40 or 41. The length of the chain was assumed to equal one, the numbering of the masses was from the left edge of the chain, the edges were assumed fixed. The system of first-order equations corresponding to system (1.6) was integrated numerically by the Runge–Kutta method with automatic sampling of the integration spacing. Assignment of the initial conditions was by a separate semiprogram. During the computation the values of $\mathbf{u}_j$ were imprinted (in graphical form) as were also those for $d\mathbf{u}_j/d\tau$ and $F(\mathbf{u}_j)$. At the end of the computation, information about the time trajectories at 10 isolated masses were stored and produced simultaneously in the form of a graph. To check the accuracy of the computation, the value of the total system energy was printed out (this quantity was conserved to 1% accuracy during the computation process).

2. Integration of system (1.6) under conditions of applicability of the long-wave approximation was performed first. As should have been expected, the computed data here are practically in agreement with the analytic solution, demonstrating the characteristic features of a soliton of an envelope under appropriate initial conditions. However, from the viewpoint of realizing intensive impact regimes, the cases of greatest interest are when the conditions mentioned are spoiled. As before, the initial conditions were given in the form corresponding to the solution of (1.5), (1.4) for $\tau = 0$ for the numerical investigation of the system (1.6) in these cases. Evolution of the initial perturbation in the case of its com-

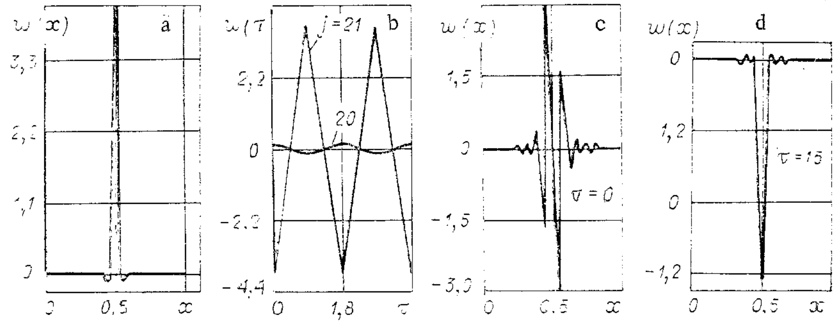

Fig. 2

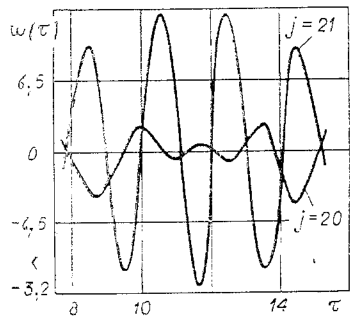

Fig. 3

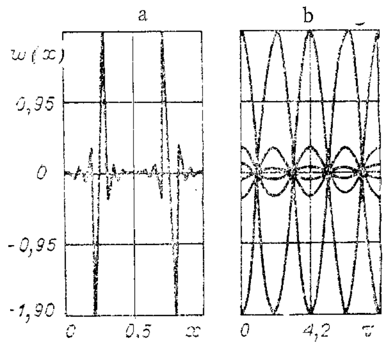

Fig. 4

paratively small amplitude is shown in Figs. 1a and 1b. The significant magnitude of its coupling parameter ($\beta = 1$) results in the fact that the time development of the process does not already follow (1.4). Nevertheless, the typical behavior of the soliton of an envelope is present, the synchronized motion of all the masses with the spatial configuration attenuating from the center of the soliton conserved. Therefore, the initial perturbation of the form

$$u_j^{(1)}(0) = u(\xi_j, 0) = \left(4/\sqrt{1+\alpha^2}\right)(-1)^j \operatorname{arth} \frac{\sqrt{\omega^2 - 1}}{\omega \operatorname{ch}\left[\sqrt{\frac{\omega^2 - 1}{\beta}} M (\xi_j - 1/2)\right]}, \tag{3.1}$$

$$\frac{du_j(0)}{d\tau} = \frac{du}{d\tau}(\xi_j, 0) = 0, \quad u_j^{(2)}(0) = \alpha u_j^{(1)}(0), \quad \frac{du_j^{(2)}(0)}{d\tau} = 0,$$

where M is the number of masses, practically corresponds, in this case, to the exact solution for the soliton of an envelope although its time evolution cannot possibly be predicted quantitatively from (1.5) and (1.4).

A systematic investigation of the influence of the coupling parameter β on the nature of the localized waves under intensive excitations of the impact type is reflected in Figs. 2 and 3. The initial conditions were given in the form of (2.1). In the weak coupling case a quite definite spatial localization of the process is observed (Fig. 2a), which is completely conserved in time. Attention is turned to the synchronization of the behavior of the strongly and weakly excited masses. The time dependence of the displacement (Fig. 2b) is characteristic for a regular impact regime. Let us note that, in this case, the analytic solution (1.5) and (1.4) is inapplicable despite the low value of β since the degree of localization is too great and the passage to the long-wave approximation (1.3) is already not justified.

Figures 2c and 2d reflect the tendency to localization in the case when the initial excitation differs somewhat from (2.1). The initial mode with two strongly excited central masses evolves in a spatially localized process in which just one mass oscillates in the impact regime. The deviation from the "exact" initial conditions for the soliton of an envelope results in the origination of an indefinite "background" that interacts with the soliton. This interaction is manifest in the quasiperiodicity of the process, the recurrence that is

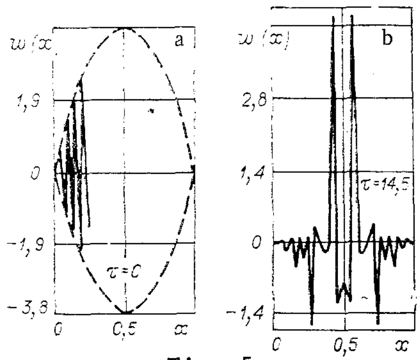

Fig. 5

seen well on the time trajectories of the masses. An analogous tendency to localization is also observed as the coupling increases further. The appropriate time graphs (Fig. 3) also reflect the influence of the "background" and demonstrate a periodic return to almost the initial relationship between the amplitudes of the different masses.

The existence of a spatially localized stationary solution makes the possibility of two-, three-soliton, etc., regimes evident. Such a possibility is illustrated in Fig. 4, where results are presented for a numerical integration of the system (1.6) under the initial conditions

$$u_j^{(1)}(0) = \frac{4}{\sqrt{1+\alpha^2}}(-1)^j \left\{ \operatorname{arth} \frac{\sqrt{\omega^2-1}}{\omega \operatorname{ch}\left[\sqrt{\frac{\omega^2-1}{\beta}} M(\xi_j - 1/4)\right]} + \operatorname{arth} \frac{\sqrt{\omega^2-1}}{\omega \operatorname{ch}\left[\sqrt{\frac{\omega^2-1}{\beta}} M(\xi_j - 3/4)\right]} \right\},$$

$$u_j^{(2)}(0) = \alpha u_j^{(1)}(0), \quad \frac{du_j^{(1)}(0)}{d\tau} = \frac{du_j^{(2)}(0)}{d\tau} = 0.$$

Conservation of the initial spatial distribution of the amplitudes in time indicates the weak interaction between the solitons.

Analysis of the numerical results shows that in the case of strong coupling and weak spatial localization, when impact interaction is not manifest, the first terms play a fundamental role in the power-law expansion of the nonlinear characteristic. Localized solutions in such a quasilinear limit were studied in [7]. The long-wave approximation (1.3) can be used here, but without neglecting the term r^2 as compared with one. If the coupling along the chain is weak, while the localization is quite definite (the impact interaction case), then the total time period is determined with good accuracy by the equation of motion of the fundamental mass performing a sawtooth oscillation.

We now discuss briefly the influence of damping. In the case of a "pure" soliton, comparatively little viscous friction will result in smooth damping of the process with the fundamental features conserved a specific time. For a certain deviation from the initial conditions corresponding to a "pure" soliton, energy pumping is observed from the main mass to the adjacent mass such that the amplitudes of these latter can even grow substantially in the initial stage of the motion.

The features described above for an envelope soliton are conserved completely even in the case when impact interaction is concentrated in a number of belts along the chain. The initial conditions were given here also in conformity with the analytic solution for an envelope soliton. The spatial and time dependences of the displacements, as well as the impact interaction forces, reflect the synchronized motion of the masses and the quite definite localization of the process.

Until now we spoke about the evolution of the initial perturbation which is similar in shape to the soliton or the multisoliton solution. Another extreme case is the excitation over nonlocalized modes, for instance over the spatial harmonic with a comparatively small number of halfwaves. Investigation of the development of the initial perturbations by means of the first, third, and tenth harmonics indicates the tendency of the system to almost "sawtooth" configurations. This latter transformation of the vibrational process (the case of the first harmonic) can be assessed from Fig. 5 in which the tendency to destruction of the "teeth" with subsequent localization is seen.

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PORE EXPANSION IN PLASTIC METALS UNDER SPALL

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The necessity to construct adequate models of material rupture under intensive dynamic loads of shock-wave nature requires a more complete comprehension of the regularities of generation and growth of individual damage during rupture. It is shown [1] that in the case of plastic metals such as aluminum and copper the damage being formed during spall is in the form of pores whose shape is almost spherical. On the basis of experimental investigations, an empirical regularity is proposed in [1] that describes the growth of an individual pore, the so-called law of viscous growth. Estimates are made in [2] for pores expanding in a plastic medium, while a kinematic model based on dislocation mechanics is proposed in [3] to describe pore growth. The model of a viscoplastic medium is used in [4] to model the spalling rupture of copper, where it is shown that satisfactory agreement between the results of experiment and computation is achieved for an extremely low value of the viscosity.

An experimental investigation of the spalling rupture of a number of metals in a broad temperature range was performed in [5]. Results of a metallographic analysis of the tested specimens, presented in [6] and subsequent papers, showed that if the viscous nature of the spalling rupture is inherent for plastic metals with fcc lattice in the whole temperature range investigated, then the viscous nature of rupture is observed under elevated test temperature conditions for metals with other types of crystalline structure. For example, characteristic spall damage in certain metals is presented in Fig. 1: a) lead, T = 0°C, P = 0.69 GPa, ×200; b) nickel, T = 0°C, P = 3.14 GPa, ×800; c) titanium alloy, VT14, T = 800°C, P = 4.25 GPa, ×500; d) Armco iron, T = 800°C, P = 2.74 GPa, ×500.

The behavior of plastic metals under intensive high-velocity plastic strain conditions is described most correctly within the framework of the model of a viscoplastic medium. In this paper the problem of examining the expansion of an isolated pore in a viscoplastic medium under the effect of a short tension pulse is posed, viz.: determine the influence of the fundamental model parameters (viscosity and yield) on the nature of spherical pore expansion. The comparison of such computed results with the results of an experimental observation of the characteristic dimensions of pores being formed during spall can be the basis for determining the viscosity of metals under spalling rupture conditions.

The problem of pore expansion in a viscoplastic medium can be formulated analogously to the problem of its collapse [7]. At a certain time let a pressure pulse P(t) be applied to the outer surface of a spherical cell of radius b_0 with inner cavity of radius a_0. We consider the material of the medium incompressible; consequently, the subsequent motion of the substance is related uniquely to the expansion of the inner cavity. The equation of motion under radial symmetry conditions has the form

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